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ELECTRICAL MEASUREMENTS AND THEIR INDUSTRIAL APPLICATIONS

HOW GOOD IS AN IRON-CORED COIL?

● WHEN MR. ARGUIMBAU WROTE HIS ARTICLE for the November, 1936, issue of the General Radio *Experimenter* on "Losses in Audio-Frequency Coils," he approached the subject from a refreshingly new point of view, and gave a number of very useful new concepts to help understand the behavior of such coils. An example is the single template which can be used to draw the curve, on log-log paper, of storage factor Q against applied frequency for *any* coil, provided only that the maximum Q of the coil and the frequency at which it occurs are known, and that resonance is remote. Alternatively, the template can be used to draw a smooth curve through a number of experimentally determined points. One such template is shown on page 2.

Employment of a useful tool like this soon makes it part of one's mental equipment. Then comes a desire to extend its usefulness by making it help answer a wider field of questions. Predictions would be very useful indicating the maximum Q of a coil and the frequency at which it occurs as changes are made in the characteristics of the iron core. For instance, how much would doubling the stack height of the iron increase Q_{max}, or what would decreasing the thickness of the laminations do to Q_{max} and f_{max}? Many other similar questions will occur.

If a theory can be provided to answer such questions, the behavior of almost any projected construction can be extrapolated from empirically obtained information. This need be less extensive than might be expected. Experimental values of Q_{max} and f_{max} for several different air gaps in an average-sized lamination would suffice, although more data would be preferable in that they would permit intercomparisons.

The extrapolated results, which must be based on certain assumptions, are necessarily approximate but still are accurate enough for most design calculations. Knowledge of the exact value of Q is rarely necessary, order of magnitude generally being sufficient.

CONDITIONS AND ASSUMPTIONS

These conditions and assumptions are as follows:

(1) Measurements are to be made at such a level that the iron has its initial permeability, that is, that the flux density, B, is vanishingly small.

(2) Under condition (1) the hysteresis loss in the iron core-material vanishes. This point is taken up in more detail on pages 11 and 12.

(3) Skin effect of the copper wire of which the coil is wound is negligible. This assumption is justified since skin effect at audio frequencies is encountered only in rather large copper wires, larger than one would be likely to employ in winding coils for use at those frequencies, such as coils for wave filters.

(4) There is negligible leakage flux traversing the copper winding. This means that eddy-current losses in the copper can be neglected. It also postulates uniform B throughout the whole magnetic path. When air gap becomes large, leakage flux is no longer negligible. Then eddy-current losses increase and the effect of the air gap on μ and on f_m cannot be calculated from simple theory.

(5) There are negligible eddy currents between adjacent laminations.

(6) Resonance is remote.

What, then, is the good of results applicable only when B is almost zero? Of course, many iron-cored coils are power transformers, operating at 10 to 12 kilogausses, some regulating types even working purposely in the saturation region at still higher flux densities. But in the communications field there are many applications for transformers and reactors operating at exceedingly low levels — for example, microphone or interstage transformers, low-level wave filters. In fact, low level is often a hindrance in the design of a transformer, because the iron permeability is so low that more turns are needed to provide the requisite minimum inductance. Even audio transformers in higher-level stages must be designed to have adequate inductance at initial permeability to prevent distortion when the audio signal drops to a very low value, such as during pianissimo orchestral passages. Furthermore, even though this analysis can be used directly only at very small B, once it is thoroughly understood it is not difficult to make estimates of the modifications required at higher levels, where the effect of hysteresis is no longer negligible. Increasing hysteresis losses decrease Q_m and mask out somewhat the contributions to Q_m, as frequency varies, of ohmic and eddy-current losses. Frequency f_m is unaffected. [See discussion accompanying Equation (20) and Figure 3 in Appendix.] Although the Q-f curve has a flatter top at higher B's, the remote wings are the same and the maximum occurs at the same frequency.

Template for theoretical relation Q vs. frequency (correct for log-log paper, 2½" per decade, K & E, No. 359-120). A cardboard template will be sent upon request.

EXPRESSIONS FOR Q_m AND f_m

If the listed conditions are met, the following expressions, taken from a detailed derivation in the Appendix, give the maximum Q of the coil and the frequency at which it occurs:

$$Q_m = \frac{1}{\delta}\sqrt{\frac{3\rho_i S A\alpha}{\rho_c t l}} \qquad (22)$$

$$f_m = \frac{10^9}{4\pi^2\mu\delta}\sqrt{\frac{3\rho_c\rho_i t l}{S A\alpha}} \qquad (23)$$

(For meanings of the symbols, consult Glossary near beginning of Appendix.) These properties (Q_m and f_m) are given in terms of dimensions of the lamination, resistivities of the copper and iron, and the permeability of the core material. Rewriting Equations (22) and (23) as follows will show more clearly the nature of the separate contributing factors:

$$Q_m = \sqrt{3} \cdot \sqrt{\frac{A\alpha}{\delta^2 l}} \cdot \sqrt{\rho_i} \cdot \sqrt{\frac{S}{t}} \cdot \sqrt{\frac{1}{\rho_c}} \qquad (22a)$$

numeric — geometry of core — material of core — geometry of coil — material of coil

$$f_m = \frac{10^9\sqrt{3}}{4\pi^2} \cdot \sqrt{\frac{l}{\delta^2 A\alpha}} \cdot \sqrt{\frac{\rho_i}{\mu^2}} \cdot \sqrt{\frac{t}{S}} \cdot \sqrt{\rho_c} \qquad (23a)$$

All of the factors determining Q_m and f_m are purely physical properties of the core and coil structure, with the single exception of the factor S, which is the effective copper winding area, and which in a sense is a derived property of the core structure.

It must be clearly understood that the permeability appearing in the formulae is the effective permeability of the path in the structure employed, which in general must be less than that of the iron obtained with ring samples. Equation (3) or (3a) of the Appendix gives an expression relating effective and true incremental permeabilities. However, there are many uncertainties in its employment. The effective length of the gap is almost never the same as the measured gap, for a variety of reasons. It is usually greater, but in the case of very large gaps may be less because of the effects of fringing. It is not completely satisfactory to regard, as some have suggested, every gap as being effectively longer than it really is by a fixed length equal to the equivalent length of a butt joint. A further complication arises from the fact that a gap in the iron leg inside the coil has more effect than one of the same length in a leg (or legs) outside the coil.

It is better, therefore, to use the empirical approach in getting the original data, the springboard from which to jump. The Q_m and f_m should be obtained for at least one core structure at a number of air gaps covering the range from complete interleaving (no gaps) to the largest practical gap. Interpolation between experimentally derived points can be done directly on the log-log plot, like Figure 4 of Mr. Arguimbau's paper, or, better still, by using an auxiliary curve (on the same sheet, if desired) of length of air gap, g, against f_m. Three such curves are the inclined, dashed ones on Figure 1. The Q_m for any one structure will be sensibly independent of the air gap. It will be found safe to predict μ on the basis of gap-length ratio (g/l); that is, if one structure has twice the l of another and twice the g, the μ will be the same for purposes of Equation (23).

FIRST DEDUCTIONS

What are the most obvious facts to be gleaned from these two expressions, Equations (22) and (23)?

1. If cores and coils are considered having similar proportions but different sizes (every dimension altered by the same factor), f_m is inversely proportional and Q_m is directly proportional to any homologous dimension. This means, for instance, that for a $1\frac{1}{2}''$-tongue lamination f_m would be $\frac{1}{2}$ as great and Q_m twice as great as for a $\frac{3}{4}''$-tongue lamination (lamination thickness being unchanged).

2. f_m and Q_m are inverse with δ, the thickness of laminations.

3. f_m is inverse with μ, which is an effective μ that takes into account the effect of any air gaps in the magnetic circuit.

4. Q_m is independent of μ and hence also of air gap.

5. f_m is independent of hysteresis loss, hence of B, the flux density.

SPECIFIC EFFECTS

Now, suppose these general observations be applied to specific problems which might be encountered in practice. What will happen, for instance, if:

1. The whole structure is changed in size but not in shape, each homologous dimension being multiplied by a factor r? Q_m increases and f_m decreases by this factor r.

2. Lamination thickness is diminished? First, δ is smaller. Also, α becomes smaller because a smaller effective amount of iron can be assembled into the coil. This is because: (a) the scale makes up a bigger proportion of the core; and (b) it is impossible to pack in the iron so tightly, since it gets too flimsy to withstand such heavy pushing forces. Since α decreases, Q_m does not increase quite inversely with δ and f_m increases slightly faster than inversely with δ, the disparity being a factor $\sqrt{\alpha}$.

3. The window is not filled with copper (S below normal)? Q_m decreases and f_m increases as the square root of the copper factor decreases, unless t also changes. (Example: What are Q_m and f_m of a transformer primary, or secondary, only?)

4. Similarly, the core is not filled with iron (α of iron less than normal)? As in 3 just above, Q_m decreases and f_m increases as the square root of the stacking factor of the magnetic material decreases.

5. The coil is wound for a higher stack of iron, the laminations having the same contour? A would increase, making Q_m increase and f_m decrease proportionally to the square root of the stack height of the iron were it not that t is increased simultaneously. This partially reduces the effect of the higher stack so that the changes in Q_m and f_m are less than proportional to the square root of the stack height. For example, let us consider, for lamination proportions usually encountered, that the stack height is changed from once to twice the width of the center leg of the lamination. In this case it has been found that Q_m and f_m change by a factor of approximately 1.25, instead of 1.41 (the square root of 2, the stack-height factor).

6. ρ_i is decreased, say by substituting A-metal for silicon-steel laminations? Q_m and f_m would decrease with the square root of ρ_i. However, in this particular case f_m would decrease still further, because the initial μ of A-metal is so much larger than that of silicon steel.

7. ρ_c is increased, say by winding the coil with resistance wire? Q_m would decrease and f_m would increase with the square root of ρ_c.

8. One or more air gaps are inserted in the magnetic circuit? Q_m would be unchanged, f_m would vary inversely with

the effective μ of the magnetic circuit (or, expressed differently, with the inductance L of any particular coil).

9. Combinations of the above changes are made? The net result will be an alteration which is measured by the product of the alterations produced by each of the individual changes.

EXPERIMENTAL CONFIRMATION

How well do the experimental facts bear out this theoretical analysis? This will be shown by three examples of varying complexity selected from the information on a chart herewith, Figure 1, similar to the one on page 4 of Mr. Arguimbau's article but containing a great deal more information subsequently obtained. In each example two different cases will be compared. The data will be presented in columnar form for greater ease in comparison. Where a ratio is used, it is expressed as the ratio of the second case to the first.

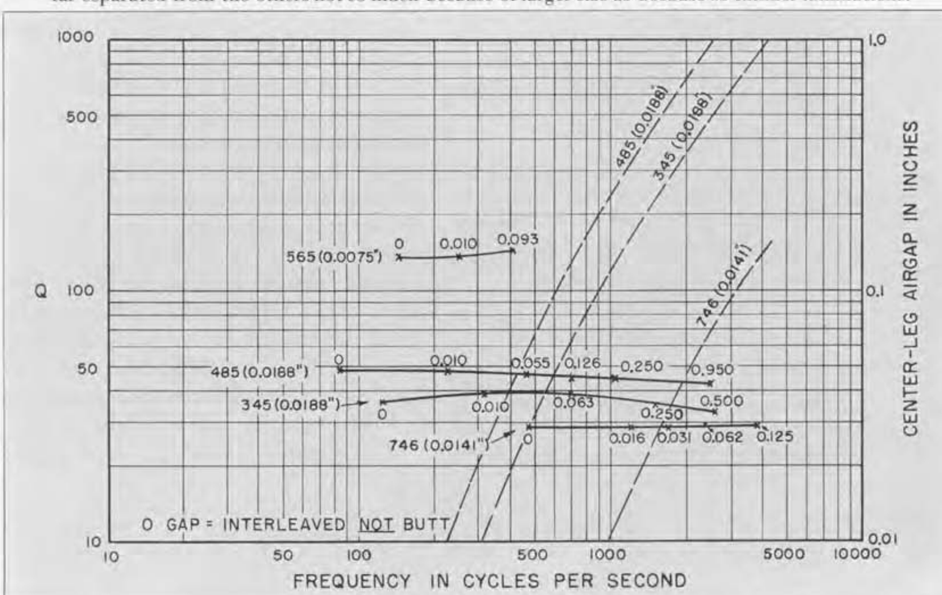
1. This is an example where coils are compared, wound on square cores using two standard General Radio laminations, of the same thickness (0.0188″), of approximately the same proportions, but of different sizes. Each magnetic circuit contains an air gap in the center leg only, proportional to the length of the magnetic circuit in each instance, which should keep the effective μ of the circuit the same.

Case	*I*	*II*
GR Type	345	485
Q_m	39	48
f_m	310 c	250 c
Air Gap	0.010″	0.0133″
Width of Center Leg	¾″	15⁄16″

The ratio of the Q_m's would be by theory the ratio of two homologous dimensions, or $\frac{15/16''}{3/4''} =$ 1.25. Compare the measured ratio: $\frac{48}{39} = 1.23$.

Similarly, the ratio of the f_m's by theory would be inverse with the homologous dimen-

FIGURE 1. Plots of Q and g vs. f.
Horizontal solid curves show locus of Q_{max} as a function of f_{max}. Curves are labeled by GR Type number of lamination, and by thickness of each lamination in parenthesis.
Inclined, dashed curves show locus of f_{max} (frequency at which Q is maximum) as a function of center-leg air gap.
Laminations progress from small to large in this order: 746, 345, 485, 565. The 565 curve is so far separated from the others not so much because of larger size as because of thinner laminations.



sions, which would be $\frac{3/4''}{15/16''} = 0.80$. The measured ratio is $\frac{250}{310} = 0.81$.

2. This example (taken from data *not* shown in Figure 1) compares Q_m and f_m of two identical coils on our TYPE 485 Core having different thicknesses of completely interleaved laminations (closest possible approach to zero air gap).

Case	*III*	*IV*
Q_m	39	93
f_m	110 c	410 c
δ	0.0192″	0.0075″
L	6.35 h	3.7 h
Weight of Iron	23.5 oz.	18.5 oz.

δ is changed by a ratio of 0.39.

α is changed by the ratio of the weights of iron, 0.787. This neglects the effect of scale on the iron, which at a thickness of 0.0075″ has not yet become an appreciable fraction of the total thickness.

However, a still further factor must be considered; μ is lower for the thin iron. It will be noted that L is changed by a ratio of 0.583, more than can be accounted for by the change in α. The difference, or a ratio of $\frac{0.583}{0.787} = 0.741$, can be ascribed to the decrease in μ. It is known that permeability does decrease for the thin, heavily-worked gauges of silicon steel.

The ratio of the Q_m's by theory would then be the reciprocal of the ratio of the δ's multiplied by the square root of the ratio of the α's, or $\frac{\sqrt{0.787}}{0.39} = 2.27$. Compare the measured ratio: $\frac{93}{39} = 2.38$.

The ratio of the f_m's by theory would be the reciprocal of the product of the ratios of the δ's and the μ's, and the square root of the ratio of the α's. This would be

$$\frac{1}{0.392 \times 0.741 \sqrt{0.787}} = 3.87.$$

The measured ratio is $\frac{410}{110} = 3.73$.

3. This is the most complicated comparison, between two coils, one using GR TYPE 345 Laminations and the other using the very small Allegheny TYPE F12 Laminations. Lamination thicknesses and stack heights vary as well as the dimensions of the laminations themselves.

Case	*V*	*VI*
Type	GR-345	F12
Q_m	38	29
f_m	980 c	2350 c
δ	0.0188″	0.0141″
Air Gap	0.111″	0.062″
Width of Center Leg	3/4″	11/32″
Stack Height	3/4″	23/32″

The ratio of the δ's is $\frac{0.0141}{0.0188} = 0.75$.

The ratio of homologous sides equals

$$\frac{11/32}{3/4} = 0.46.$$

In addition to these factors, others are necessitated by the extra stack height of the F12 lamination; ratio of A to that of a square-center-leg stack equals $\frac{23/32}{11/32} = 2.09$. The ratio of t's = 1.34.

The ratio of the Q_m's by theory equals $\frac{1}{0.75}(0.46)\sqrt{\frac{2.09}{1.34}} = 0.765$. Compare the measured ratio: $\frac{29}{38} = 0.764$.

The ratio of f_m's by theory equals

$$\frac{1}{0.75} \times \frac{1}{0.46}\sqrt{\frac{1.34}{2.09}} = 2.32.$$

The measured ratio is $\frac{2350}{980} = 2.40$.

CONCLUSIONS

In all of the above comparisons it has been assumed that the laminations are strictly similar in shape, which is not exactly true. However, the reasonably good agreement between the theory and the actual measurements for a number of different sizes of laminations can logically be taken to indicate, first, that the theory is adequate and, second, that small departures of the lamination dimensions from strict similarity do not have any major effect on the results.

It is, therefore, apparent that the use of Equations (22) and (23) will yield, with satisfactory approximation, a good picture of the behavior of a particular proposed coil structure, provided there is a small amount of reliable information on which to base the predictions.

HOW TO USE

To discover and put to use any extrapolated information such as has been described, proceed as follows:

1. List all of the properties and dimensions which differ for the two cases to be compared. Values for l will be needed if gap lengths g are to be altered to keep μ unchanged.

2. Calculate Q_m and f_m for the new structure from the known corresponding values of the old structure and the information in 1.

3. Plot the point corresponding to the new Q_m and f_m on the log-log paper. Lay the template on the paper with the long straight side parallel to the f-axis and with the (marked) center of the hump of the curve at the point just plotted. The behavior of *any* coil wound on this structure over a wide range of frequencies will be shown by the template [barring, of course, skin effect (very large wires), resonance (very high inductance), or other anomalous circumstance]. The curve may be actually drawn using the template, or, if it would cause confusion, on a sheet bearing a great deal of information, values could be read directly from the edge of the template.

APPENDIX

Losses in an iron-cored coil come about from four sources: namely, I^2R loss and eddy-current loss in the copper, hysteresis and eddy-current loss in the iron. Eddy-current losses in the copper will be ignored in this analysis for two reasons. The first is that the audio frequencies considered will be too low and/or the wire sizes too small to have appreciable eddy-current loss. The second, and more important reason, is that the iron core effectually prevents most of the flux from traversing the window in which the copper of the coil is located.

GLOSSARY

The symbols used are tabulated next, with their definitions and dimensions.

E = r-m-s alternating emf across coil; volts $(= I\omega L)$.

I = r-m-s alternating current through coil; amperes.

L = inductance of coil; henrys.

f = frequency of alternating voltage and current.

$\omega = 2\pi f$.

$\mathfrak{F}$ = r-m-s magnetomotive force; gilberts.

H = r-m-s magnetic force produced by current I; oersteds.

B = r-m-s flux density within the iron; gausses.

B_m = max. instantaneous value of alternating flux density $= B\sqrt{2}$; gausses.

Φ = total r-m-s flux in the iron; maxwells.

$\mathfrak{R}$ = reluctance of magnetic path.

P_h, P_e = power dissipated in the iron by hysteresis and eddy currents, respectively; watts.

R_h, R_e = equivalent resistances corresponding to P_h and P_e; ohms.

R_c = ohmic resistance of the copper in the coil; ohms. (Assumed the same as the d-c value; that is, no skin effect.)

D_c, D_h and D_e = dissipation factors corresponding to R_c, R_h, and R_e, obtained by relating each equivalent resistance to the coil reactance ωL; dimensionless.

D = total dissipation factor = sum of D_c, D_h, and D_e.

ρ_c = resistivity of copper; ohm-cm.

d = wire diameter (exclusive of insulation); cm.

T = total length of copper wire; cm.

t = length of average turn; cm.

N = number of turns of wire.

$S = N \cdot \frac{\pi d^2}{4}$ = effective window area (total copper cross section); cm^2.

ρ_i = resistivity of the lamination material; ohm-cm.

δ = lamination thickness; cm.

A = total geometric cross section of magnetic path; cm^2.

α = stacking factor of iron; dimensionless (ratio of effective area of core material to inside area of coil tube; deficiencies are occasioned by scale, burrs, bent laminations, core-plating, etc.).

l = mean length of flux path; cm.

V = volume of magnetic material = $lA\alpha$; cm^3.

g = total length of air gaps; cm.

μ = incremental permeability (effective) of magnetic circuit.

μ_t = incremental permeability (true, ring-sample) of magnetic material.

η = hysteresis constant.

ϵ = Steinmetz exponent.

DERIVATION

Most of the basic equations given below can be found in any textbook or handbook of electricity. The first six define $\mathfrak{F}$, $\mathfrak{R}$, μ, Φ and L in terms of coil parameters and current through the coil:

$$\mathfrak{F} = \frac{4\pi NI}{10} \qquad (1)$$

$$\mathfrak{R} = \frac{l-g}{\mu_t A\alpha} + \frac{g}{A} = \frac{1}{A\alpha}\left(\frac{l-g}{\mu_t} + g\alpha\right) = \frac{l}{\mu A\alpha} \qquad (2)$$

where

$$\mu = \frac{\mu_t}{1 + \frac{g}{l}(\mu_t\alpha - 1)} \qquad (3)$$

and, since usually $\mu_t\alpha \gg 1$, approximately

$$\mu = \frac{\mu_t}{1 + \frac{g}{l}\mu_t\alpha} \qquad (3a)$$

$$\Phi = \frac{\mathfrak{F}}{\mathfrak{R}} = \frac{4\pi NI\mu A\alpha}{10l} \qquad (4)$$

$$\Phi = BA\alpha \qquad (5)$$

Also, using Equation (4):

$$L = \frac{\Phi N}{10^8 I} = \frac{4\pi N^2\mu A\alpha}{10^9 l} \qquad (6)$$

COPPER LOSS

The series ohmic resistance of the coil is given, from resistivity, by:

$$R_{c(ser)} = \rho_c \frac{T}{\frac{\pi}{4}d^2} = \frac{4\rho_c Nt}{\pi d^2} \qquad (7)$$

The dissipation factor corresponding to this can be reduced by the use of Equation (6):

$$D_c = \frac{R_{c(ser)}}{\omega L} = \frac{4\rho_c N t}{\pi d^2} \cdot \frac{10^9 l}{2\pi f 4\pi N^2 \mu A \alpha}$$
$$= \frac{10^9 l \rho_c t}{8\pi^2 f N d^2 \mu A \alpha} = \frac{10^9 \rho_c t l}{8\pi^2 f \mu S A \alpha} = \frac{c}{f} \quad (8)$$

This dissipation factor is found to be inversely proportional to frequency, the factor of proportionality being:

$$c = \frac{10^9 \rho_c t l}{8\pi^2 \mu S A \alpha} \quad (9)$$

HYSTERESIS LOSS

The power expended in hysteresis loss is given by (since $B_m = B\sqrt{2}$):

$$P_h = \eta V f B_m^{\epsilon} 10^{-7} = \eta V f 2^{\epsilon/2} B^{\epsilon} 10^{-7} \quad (10)$$

Since power equals $\frac{E^2}{R}$, the equivalent parallel resistance of the hysteresis loss is, using also Equations (5) and (6):

$$R_{h(par)} = \frac{E^2}{P_h} = \frac{I^2\omega^2 L^2}{P_h}$$
$$= \frac{4\pi^2 N^2 B^{(2-\epsilon)} f A \alpha}{2^{\epsilon/2} 10^9 \eta l} \quad (11)$$

Similarly, the equivalent series resistance is:

$$R_{h(ser)} = \frac{2^{\epsilon/2} 16\pi^2 \eta \mu^2 N^2 A \alpha f}{10^9 B^{(2-\epsilon)} l} \quad (11a)$$

The corresponding dissipation factor, reduced by Equation (6), is:

$$D_h = \frac{\omega L}{R_{h(par)}}$$
$$= \frac{2\pi f 4\pi N^2 \mu A \alpha}{10^9 l} \cdot \frac{2^{\epsilon/2} 10^9 \eta l}{4\pi^2 N^2 B^{(2-\epsilon)} f A \alpha}$$
$$= \frac{2^{\epsilon/2} 2\eta\mu}{B^{(2-\epsilon)}} = h \quad (12)$$

This factor is independent of frequency and has the value:

$$h = 2^{(1+\epsilon/2)} \eta \mu B^{(\epsilon-2)} \quad (12a)$$

EDDY-CURRENT LOSS (IRON)

The power expended in eddy-current loss in the iron is given by:

$$P_e = \frac{\pi^2 f^2 B_m{}^2 \delta^2 V}{6 \times 10^{16} \rho_i} = \frac{\pi^2 f^2 2 B^2 \delta^2 V}{6 \times 10^{16} \rho_i} \quad (13)$$

Since power $= \frac{E^2}{R}$, the equivalent parallel resistance, reduced by Equations (5) and (6), is:

$$R_{e(par)} = \frac{E^2}{P_e} = \frac{I^2\omega^2 L^2}{P_e} = \frac{12\rho_i A \alpha N^2}{\delta^2 l} \quad (14)$$

The corresponding dissipation factor, reduced by Equation (6), is:

$$D_e = \frac{\omega L}{R_{e(par)}} = \frac{2\pi f 4\pi N^2 \mu A \alpha}{10^9 l} \cdot \frac{\delta^2 l}{12\rho_i A \alpha N^2}$$
$$= \frac{2\pi^2 \delta^2 \mu f}{3\rho_i 10^9} = ef \quad (15)$$

This dissipation factor is directly proportional to frequency, and the factor of proportionality is:

$$e = \frac{2\pi^2 \delta^2 \mu}{3\rho_i 10^9} \quad (16)$$

EQUIVALENT CIRCUIT REPRESENTING LOSSES

It is interesting to note how this analysis demonstrates the correctness of the usual method of showing the equivalent circuit of an iron-cored coil or transformer, as in Figure 2. Here R_c and R_e are resistances independent of frequency, representing respectively ohmic loss in the copper and eddy-current loss in the iron. Equations (7) and (14) show the invariance of R_c and R_e with frequency. R_h, on the other hand, whether calculated as a series or as a parallel resistance [Equations (11) and (11a)], varies with the first power of frequency. Hysteresis loss, therefore, cannot be represented as a resistance independent of frequency and hence is not shown in the equivalent circuit of Figure 2.

TOTAL LOSS

The total dissipation factor, D, is the sum of the three separate dissipation factors:

$$D = \frac{c}{f} + h + ef \qquad (17)$$

OPTIMUM CONDITIONS

When plotted on log-log paper each component is a straight line as shown in Figure 3, that for hysteresis being horizontal and those for copper and eddy current being slanted down and up at 45°, respectively. Minimum D occurs where the c and e lines cross, at a frequency given by

$$f_m = \sqrt{c/e} \qquad (18)$$

At this frequency the minimum D is

$$D_m = h + 2\sqrt{ce} \qquad (19)$$

When a curve of D for any coil has been found experimentally, numerical values for the three coefficients c, h, and e, can be found by drawing 45° asymptotes to the curve. The intercepts of these lines with the 1-cycle axis are the values of

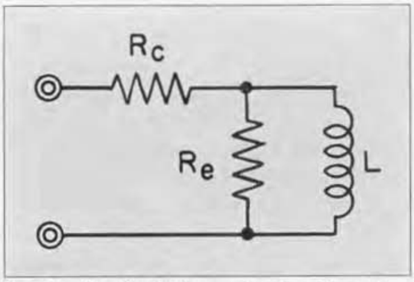


FIGURE 2. Equivalent circuit of a low-frequency coil.

c and e. The value of h is the difference between the observed minimum and twice the value of the two asymptotes at their crossing point.

Q—STORAGE FACTOR

Engineers in the radio and audio fields are more accustomed to think in terms of Q, the reciprocal of D, than in terms of D. Unfortunately, the expression just developed gives:

$$Q_m = \frac{1}{h + 2\sqrt{ce}} \qquad (20)$$

This is easily enough calculated in a given case, but it does not lend itself readily to quick mental calculations because of the presence of h. However, there is a fortunate circumstance which makes neglect of h in this expression allowable.

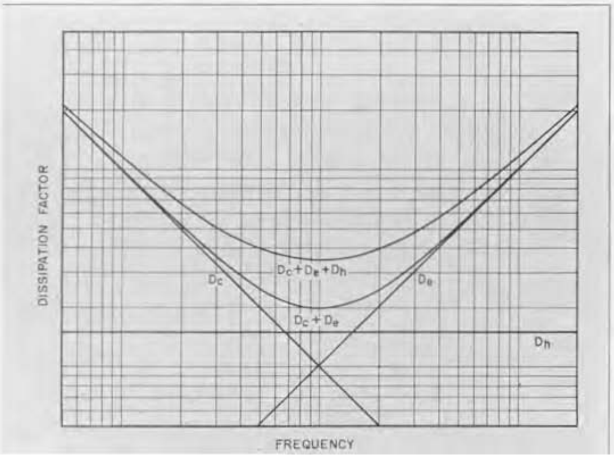


FIGURE 3. Dissipation factor-frequency relationships.

Separate dissipation factor curves are the three labeled straight lines.

At initial permeability ($D_h = 0$), the lower curve "$D_c + D_e$" represents coil behavior. Invert it to secure a Q-curve.

Above initial permeability ($D_h \neq 0$), the upper curve "$D_c + D_e + D_h$" is representative of the blunting action of a finite D_h. f_{max}, although less easily determined, is unchanged

SIMPLIFICATION OF EXPRESSION FOR Q

The factor, h, Equation (12a), contains a term $B^{(\epsilon-2)}$. All of the measurements which we have described in Mr. Arguimbau's article and in this one have been made with such a low flux density in the iron that the initial permeability plateau has been reached. This, for high-silicon steels, is in the region of B below one gauss, or, really, a place where B is approaching zero. It is very helpful to make inductance measurements on this plateau, since unavoidable small changes in the supply voltage make imperceptible changes in the permeability and, hence, the inductance. Contrariwise, once the B has become large enough so that permeability has begun to increase, it is no longer possible to have μ independent of the effects of voltage applied to a coil having a core of ferro-magnetic material (with the notable exception of some dust cores). The initial permeability plateau is the most easily reproducible measuring condition and its only drawback is the high gain required ahead of the detector in a measuring circuit. One can always be sure that he is on this plateau when making measurements by continuously reducing the voltage applied to the coil until successive reductions make no further change in the measured value.

A curve is shown in Figure 4 of the relation, at very low magnetizing forces, between H and μ for a high-silicon steel, reproduced by permission from a paper by H. W. Lamson.* This shows the plateau just referred to, a phenomenon not widely known. Note that the curve starts to ascend from the plateau ($\mu_t = 470$) at about $H = 0.001$ oersted, which corresponds to a $B(=\mu_t H)$ of about 0.5 gauss. This will indicate the order of smallness of excitation to reach the plateau for a 4% silicon steel.

If the customarily used value of ϵ, namely, the 1.6 figure of Steinmetz, holds good for these very low flux densities, then h becomes infinite, since B (approaching zero) goes into the denominator. This, we know physically, is not true, and there is implicit corroboration in the three examples of this article. However, there is a better authority than this. It is well, but not generally, known that the Steinmetz exponent is not a constant at all, but varies with flux density B and happens to have a value very close to 1.6 in the middle region, say between 2 and 10 kilogausses. On the other hand, the value of this exponent increases for both very high and very low flux densities. Values as high as three or more can readily be found in the literature for

*Proc. I.R.E., Vol. 28, No. 12, p. 546 (Dec., 1940).

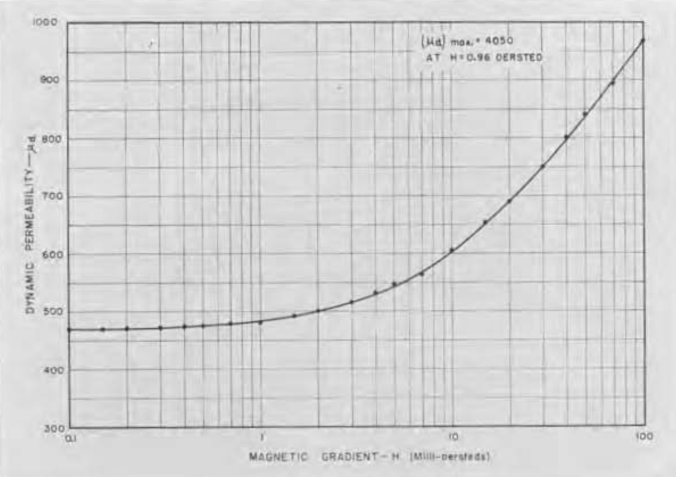


FIGURE 4. Permeability of silicon steel at low inductions, showing the initial-permeability plateau.

very high values of B,† and an exponent of 2.4 for very low B values is given.†† Further, Rayleigh, in 1887, showed that $\epsilon = 3$ at low flux densities, and this has been confirmed recently by Elwood.‖

If the Steinmetz exponent in the initial permeability region is greater than 2, then h approaches zero as B approaches zero. This means that, in the initial permeability region, h can be neglected and

$$Q_m = \frac{1}{2\sqrt{ce}}. \qquad (21)$$

FINAL EXPRESSIONS FOR Q_m AND f_m

Substituting the values for c and e from Equations (9) and (16):

$$Q_m = \frac{1}{\delta}\sqrt{\frac{3\rho_i S A \alpha}{\rho_c t l}} \qquad (22)$$

$$f_m = \frac{10^9}{4\pi^2 \mu \delta}\sqrt{\frac{3\rho_c \rho_i t l}{S A \alpha}} \qquad (23)$$

EFFECTS OF TEMPERATURE

The variation of Q_m and f_m with temperature can be calculated using the temperature coefficients of copper and iron resistivities. ρ_c has a temperature coefficient of +0.4% per degree C, while ρ_i has one of +0.5%. Temperature coefficient of c is then +0.4%, of e −0.5%. D_m (or Q_m) is sensibly constant with temperature, but f_m increases about 0.5% per degree C.

†Spooner, "Properties and Testing of Magnetic Materials" (1927), p. 24.

††Page 325 of Vol. 2 of the Dictionary of Applied Physics, 1922 Edition, quoting an article by A. Campbell, "Magnetic Properties of Stalloy in Weak Alternating Fields," Phys. Soc. Proc. 1920, XXXII, 232. (Stalloy is an English high-silicon steel.)

‖Physics 6, 215, 1935.

NOTES ON EXPRESSIONS (22) AND (23)

Although this article concerns itself only with using the equations for comparative, not absolute, purposes, the Expressions (22) and (23) will determine Q_m and f_m from constants of the core structure. This has actually been done and fairly good agreement with measured values obtained.

For example, Q_m for GR-345 core (¾″-tongue) calculates 37.7 and measures from 33 up to 40. Also, for GR-485 core (1 5/16″-tongue) Q_m calculates 49.5, measures 43 to 48. In each case the very low, disagreeing, measured values occur at high frequencies (large air gaps) where the uncertainties of Q measurements increase and where there is more probability of eddy-current losses in the copper because of fringing.

Calculations of f_m in cycles per second using as a basis the plateau μ_i of 470 (from Figure 4) in Equation (3a) are tabulated below:

Center-Leg	*f_m-345 Coil*		*f_m-485 Coil*	
Gap	*Calc.*	*Meas.*	*Calc.*	*Meas.*
Interleaved	97	122	74	84
1/16″	631	700	379	480
⅛″	1164	1050	685	700
¼″	—	—	1294	1040
½″	4360	2600	—	—
0.95	—	—	4700	2500

It will be noted that this table bears out the statement made under heading EXPRESSIONS FOR Q_m AND f_m early in the paper that the equivalent air gap is almost never the same as the measured gap, being larger than the measured value for small gaps and smaller for large gaps (larger gap means smaller μ, which means larger f_m). The μ for interleaved laminations is indicated to be slightly less than that for a ring-sample (truly gapless) material. Contrariwise, for the largest illustrative gaps in these examples the μ is larger than one would be led to expect by theory.

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